Inference in High-Dimensional Panel Models:
Two-Way Dependence and Unobserved Heterogeneity

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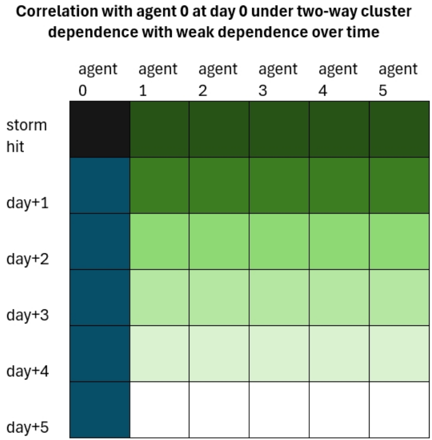
Table of Contents

Motivation: Why High Dimensionality Matters in Economics and Panel Models?

- High dimensionality: a **large number of unknown parameters**.
- Three common scenarios:
  - **Many potentially relevant variables**: e.g., provisions in trade agreements, price of relevant goods.
  - **Nonparameric or semiparametric modeling**: example
  - **Unobserved heterogeneity**: fixed or correlated random effects in nonlinear models.
- Existing high-dim. methods may not be valid for **panel data models**: estimation and inference under **two-way cluster-dependence**.

Graphic illustration of two-way cluster dependence



Preview of Results

- **Model**: a high-dimensional (regression) model for panel data. E.g., $Y_{it} = \theta_0 D_{it} + g_0(X_{it}, c_i, d_t) + U_{it}$.
- **Target**: inference for low-dim. parameters in the presence of high-dim. nuisance parameters.
- **Challenges**: unit and time cluster dependence as well as weak dependence across clusters; unobserved heterogeneity.
- **Main contribution i**: a variant of (post) **LASSO**, robust to **two-way cluster-dependence** in panel data.
- **Main contribution ii**: a clustered-panel **cross-fitting** approach.

Preview of Results

- Both the variant of LASSO and panel cross-fitting are of **independent interest**.
- Together, they allow for **consistent estimation** and **valid inference** about the low-dim. parameter.
- **Main contribution iii**: generalized-Mundlak (correlated random effects) approach in the partial linear model.
- **Application**: hidden dimensionality in estimating government spending multiplier.

Example: Hidden High Dimensionality

- **Estimation of the multiplier**: the percentage increase in output that results from the 1 percent increase in government spending.
- Researchers often start with a **baseline** model:

$$Y_{it} = \theta_0 D_{it} + X_{it}\pi_0 + c_i + d_t + U_{it},\ \mathrm{E}[Z_{it}U_{it}] = 0$$

- **Robustness check**: to avoid endogeneity caused by potential misspecification,

$$Y_{it} = \theta_0 D_{it} + g_0(X_{it}, c_i, d_t) + U_{it}.$$

- **Cost**: noisy or infeasible estimation with limited sample sizes (51 states with 39 periods).

Table of Contents

Challenge One

- To reduce dimensionality: sparse method, regularized estimator, e.g. LASSO.
- Focus on a simplified model using the pooled panel:

$$\begin{aligned} Y_{it} =& \theta_0 D_{it} + g_0(X_{it}) + U_{it} \\ =& \theta_0 D_{it} + f_{it}\beta_0 + r_{it} + U_{it} \text{ by sparse approximation} \end{aligned}$$

- Obtain $(\tilde{\theta}, \tilde{\beta})$ by running penalized least square of Y_{it} on (D_{it}, f_{it}).

Twoway Clustering Dependence in Panel

- **Assumption 1** Random elements $W_{it} = (Y_{it}, X_{it}, V_{it})$ are generated by the underlying process:

$$W_{it} = \mu + h(\alpha_i, \gamma_t, \varepsilon_{it}), \quad \forall i \geq 1, t \geq 1,$$

where $\mu = E[W_{it}]$; h is unknown; vector components $(\alpha_i)_{i\geq 1}$, $(\gamma_t)_{t\geq 1}$, and $(\varepsilon_{it})_{i\geq 1, t\geq 1}$ are mutually independent; α_i is i.i.d across i, ε_{it} is i.i.d across i and t, and γ_t is strictly stationary.
  - Common in cluster-robust inference literature.
- **Assumption 2** (*beta*-mixing of $\{\gamma_t\}_{t>1}$)
  - A generalization of Aldous-Huber-Kallenberg (AHK) representation (Chiang et al., 2024, REStat, Chen and Vogelsang, 2024, JoE).

Existing Approaches and My Proposal

- Approach 1: Assuming the stochastic error is **conditionally normal** (Bickel et al., 2009, AOS).
- Approach 2: **Self-normalizing** the non-Gaussian errors (Belloni et al., 2012, ECTA, Belloni et al., 2016, JBES)
- Approach 3: Deriving **concentration inequalities** allowing for dependent error process (Babii et al., 2023, JOE, Gao et al., 2024, WP).
- **My proposal**: Hoeffding-type decomposition; regressor-specific penalty **weights robust to two-way dependence**.
- My construction of penalty level and weights.

Consistency and convergence rate results

- **Theorem**: Given the AHK approximate sparsity, feasible weights, and regularity conditions, with some $C_\lambda = O(1)$ and $\gamma = o(1)$, we have the number of selected regressors be $O(s)$ and the l^2 rate of convergence for the (post) two-way cluster-LASSO is $O_P\left(\sqrt{\frac{s\log(p/\gamma)}{N\wedge T}}\right)$.
- **Comparison**: $O_P\left(\sqrt{\frac{s\log p}{NT}}\right)$ under random sampling as in Bickel et al., 2009, AOS; $O_P\left(\sqrt{\frac{s\log(p\vee NT)}{NT}}\right)$ under random sampling in Belloni et al., 2012, ECTA; $O_P\left(\sqrt{\frac{s\log(p\vee NT)}{Nl_T}}\right)$ under cross-sectional independence in Belloni et al. (2016) where $l_T \in [1, T]$.
- Oracle case

Table of Contents

Challenge Two

- Consider a semiparametric approach:

$$\widehat{\theta} = \left[\sum_{i=1}^{N}\sum_{t=1}^{T} D_{it}' D_{it}\right]^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T} D_{it}'(Y_{it} - f_{it}\widehat{\zeta}).$$

- $\widehat{\zeta}$ can be noisy due to two-way cluster dependence and high dimensionality.
- **A better second-step** estimator: Let $\ddot{D}_{it} := D_{it} - \widehat{\mathrm{E}}[D_{it}|X_{it}]$,

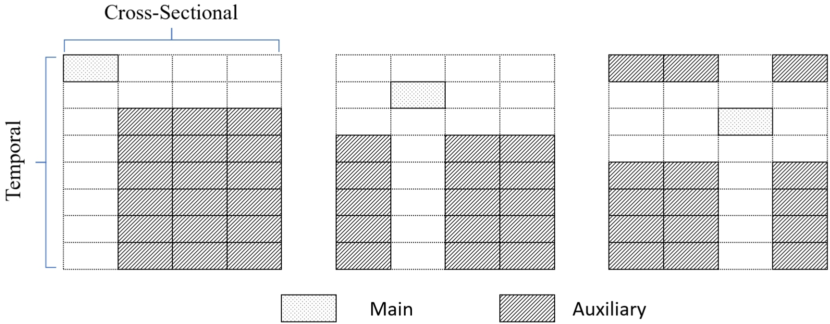
$$\widehat{\theta} = \left[\sum_{i=1}^{N}\sum_{t=1}^{T} \ddot{D}_{it}' D_{it}\right]^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T} \ddot{D}_{it}'\left(Y_{it} - f_{it}\widehat{\zeta}\right).$$

- But there is still a **problematic error term** in $\widehat{\theta} - \theta_0$:

$$\sum_{i=1}^{N}\sum_{t=1}^{T} V_{it}^{D} f_{it}\left(\zeta_0 - \widehat{\zeta}\right), \; V_{it}^{D} := D_{it} - \mathrm{E}[D_{it}|X_{it}].$$

- **Cross-fitting:** split the sample for the two-step estimations.

Clustered-Panel Cross-Fitting



Lemma (validity of the cross-fitting): Under Assumptions 1 (AHK) and 2 (beta-mixing), the cross-fitting sub-samples are "approximately" independent as $N, T \to \infty$ with $\log(N)/T \to 0$.

Asymptotic Normality

- **Theorem**: Given rates of convergence for the first-step and regularity conditions, $\sqrt{N \wedge T}\left(\widehat{\theta} - \theta_0\right) \Rightarrow \mathrm{N}(0, V)$ where $V := A_0^{-1} \Omega A_0^{-1\prime}, \ \Omega := \Lambda_a \Lambda_a' + c \Lambda_g \Lambda_g'$.
- A sufficient L^2 rate of convergence for η_0 is $o\left((N \wedge T)^{-1/4}\right)$.
- Cluster-robust variance estimators

Table of Contents

Challenge Three

- Consider the following partial linear model:

$$Y_{it} = D_{it}\theta_0 + g(X_{it}, c_i, d_t) + U_{it}, \ \mathrm{E}[U_{it}|X_{it}, c_i, d_t] = 0.$$

- Z_{it} has the same dimension of D_{it}; $\mathrm{E}[Z_{it}U_{it}] = 0$. As a special case, $Z_{it} = D_{it}$.
- In the running example, Y_{it} is the state gross output; D_{it} state military spending; X_{it} are low-dimensional controls; Z_{it} is a Bartik IV.
- Instead of imposing the separability, we consider g as an approximately sparse function and let data decide on the selection.
- (c_i, d_t) as correlated random effects .

CRE approach: the generalized Mundlak device

- A generalized Mundlak device:

$$c_i = h_c(\bar{F}_i, \epsilon_i^c), \tag{1}$$

$$d_t = h_d(\bar{F}_t, \epsilon_t^d), \tag{2}$$

  where $\bar{F}_i = \frac{1}{T}\sum_{t=1}^{T} F_{it}$, $\bar{F}_t = \frac{1}{N}\sum_{i=1}^{N} F_{it}$, $F_{it} := (D_{it}, X_{it}')'$; h_c and h_d are unknown functions; $(\epsilon_i^c, \epsilon_t^d)$ are independent shocks.

- Generalized by a flexible function. Also see Wooldridge and Zhu, 2020, JBES.
- Almost ready but there is one more subtle issue.

A Subtle Issue

- Fixed-effect and random-effect approaches **may not be compatible** with cross-fitting.
- E.g., the proxies $1/N \sum_{i=1}^{N} X_{it}$ and $1/T \sum_{s=1}^{T} X_{js}$ must share the data point X_{jt}.
- In this case, to quantify the impact on the coupling result is tricky and may require extra conditions.
- Without cross-fitting, it is generally hard to establish inferential theory with **growing dimensions**.
- It turns out inference using **full sample** is possible in this setting, under a slightly stronger sparsity condition.

inference with full sample

Government Spending Multiplier: Baseline Method

Table 1: Multiplier estimates of the original model

(1) Unobs. Heterog.	(2) Oil Price	(3) Real Int.	(4) Pop.	(5) First Step	(6) IV 1 $\widehat{\theta}$	(7) Two-way Robust s.e.
	No	No	No	POLS	1.43	0.68
	Yes	No	No	POLS	1.30	0.56
Fixed Effects	No	Yes	No	POLS	1.40	0.57
	Yes	Yes	No	POLS	1.27	0.45
	Yes	Yes	Yes	POLS	1.36	0.43

Government Spending Multiplier: Full-Sample Method

Table 2: Multiplier estimates of the extended model.

(1) Cross-Fitting	(2) Poly. Trans.	(3) Param. Gen.	(4) First Stage	(5) Z: Param. Sel.	(6) $\widehat{\theta}$	(7) Two-way Robust s.e.
No	None	7	POLS	7	1.51	0.66
			H LASSO	2	1.43	0.66
			C LASSO	4	1.43	0.66
			TW LASSO	2	1.43	0.70
No	2nd	35	POLS	35	1.73	0.99
			H LASSO	6	1.73	1.01
			CR LASSO	5	1.75	1.02
			TW LASSO	4	1.43	0.61
No	3rd	119	POLS	119	2.20	1.19
			H LASSO	10	1.97	1.16
			CR LASSO	6	0.98	0.66
			TW LASSO	6	1.47	0.59

Cross-Fitting Method Simulation

Table of Contents

Summary

- The inferential theory for high-dim. models is particularly **relevant in panel settings**.
- This paper **enriches the toolbox** of researchers in dealing with **high-dim. panel models**.
- I develop a **LASSO-based** estimator for a high-dimensional regression model and valid **inference with or without cross-fitting**.
- **Unobserved heterogeneity** complicates the inference. I propose a simple and flexible **correlated random effect** approach.
- I illustrate in a panel data application that **high dimensionality can be hidden** and how proposed approaches allow for a **robustness check**.

Thank you for listening and comments!

Simulation: DGP(i)

- DGP(i) - Linear model:

$$\begin{aligned} Y_{it} = &\ D_{it}\theta_0 + X_{it}\beta_0 + U_{it}, \\ D_{it} = &\ X_{it}\pi_0 + V_{it}, \end{aligned}$$

  where β_0 and π_0 are sparse in a cut-off design.

- DGP(i) - Additive components:

$$\begin{aligned} X_{it,j} = &\ w_1\alpha_{i,j} + w_2\gamma_{t,j} + w_3\varepsilon_{it,j}, \\ U_{it} = &\ w_1\alpha_i^u + w_2\gamma_t^u + w_3\varepsilon_{it}^u, \\ V_{it} = &\ w_1\alpha_i^v + w_2\gamma_t^v + w_3\varepsilon_{it}^v, \end{aligned}$$

Simulation: DGP(ii)

- DGP(ii) - Partial linear model:

$$Y_{it} = D_{it}\theta_0 + (X_{it}\beta_0 + c_i + d_t)^2 + U_{it},$$
$$D_{it} = \frac{\exp(X_{it}\pi_0)}{1+\exp(X_{it}\pi_0)} + V_{it},$$
$$c_i = \bar{D}_i + \bar{X}_i\xi_0 + \epsilon_i^c, \quad d_t = \bar{D}_t + \bar{X}_t\zeta_0 + \epsilon_t^d,$$

where β_0, π_0, ξ_0, and ζ_0 are sparse in a polynomial-decay design;

- DGP(ii) - Multiplicative components:

$$X_{it,j} = w_1\alpha_{i,j} + w_2\gamma_{t,j} + w_3\varepsilon_{it,j},$$
$$U_{it} = \frac{w_4}{c_p}\sum_{j=1}^{p}[\alpha_i^u\gamma_{t,j} + \alpha_{i,j}\gamma_t^u] + w_5\varepsilon_{it}^u,$$
$$V_{it} = \frac{w_4}{c_p}\sum_{j=1}^{p}[\alpha_i^v\gamma_{t,j} + \alpha_{i,j}\gamma_t^v] + w_5\varepsilon_{it}^v,$$

Simulation results

Table 1: DGP(i) with $N = T = 25$, $s = 5$, $p = 200$, $\iota = 0.5$, $\rho = 0.5$, $c_\beta = c_\pi = 0.5$

Cross Fitting	First-Step Estimator	First-Step Ave. Sel. Y	Sel. D	Second-Step Bias	SD	RMSE	Coverage (%) CHS	DKA
No	POLS	200	200	0.003	0.053	0.053	78.9	95.1
	H LASSO	26.0	26.0	0.062	0.065	0.090	58.5	78.7
	R LASSO	17.6	17.6	0.070	0.067	0.097	65.2	79.5
	C LASSO	8.6	8.9	0.036	0.095	0.101	80.0	87.5
	TW LASSO	6.7	6.9	0.023	0.096	0.099	84.3	90.4
Yes	POLS	200	200	0.006	0.113	0.113	98.2	99.4
	H LASSO	16.9	16.6	0.053	0.131	0.141	96.0	97.6
	R LASSO	9.5	9.5	0.054	0.130	0.141	96.0	98.2
	C LASSO	8.0	8.1	0.041	0.130	0.136	96.2	97.4
	TW LASSO	6.7	6.4	0.057	0.126	0.138	95.8	97.2

Simulation results

Table 2: DGP(i) with $N = T = 25$, $s = 5$, $p = 600$, $\iota = 0.5$, $\rho = 0.5$, $c_\beta = c_\pi = 0.5$

Cross Fitting	First-Step Estimator	First-Step Ave. Sel. Y	First-Step Ave. Sel. D	Second-Step Bias	Second-Step SD	Second-Step RMSE	Coverage (%) CHS	Coverage (%) DKA
No	POLS	600	600	0.008	0.221	0.221	26.6	38.6
	H LASSO	39.5	39.8	0.073	0.049	0.087	51.2	78.9
	R LASSO	25.1	25.3	0.079	0.055	0.097	52.4	79.1
	C LASSO	14.0	15.2	0.058	0.096	0.112	68.8	78.4
	TW LASSO	6.9	7.5	0.033	0.098	0.103	81.6	88.1
Yes	H LASSO	24.8	24.7	0.056	0.134	0.146	94.5	98.4
	R LASSO	12.1	12.1	0.054	0.137	0.147	94.5	96.1
	C LASSO	10.7	11.6	0.043	0.139	0.145	95.1	96.1
	TW LASSO	6.8	7.6	0.065	0.140	0.154	90.7	95.1

Simulation results

Table 3: DGP(ii) with $N = T = 25$, $s = p = 10$, $\iota = 0.5$, $\rho = 0.5$, $c_\beta = 1, c_\pi = 4, c_\xi = c_\zeta = 1/4$; 2nd-order polynomial series are used for approximation

Cross Fitting	First-Step Estimator	First-Step Ave. Sel. Y	First-Step Ave. Sel. D	Second-Step Bias	Second-Step SD	Second-Step RMSE	Coverage (%) CHS	Coverage (%) DKA
No	POLS	560	560	0.012	0.173	0.173	54.4	67.4
	H LASSO	12.2	3.4	0.032	0.126	0.130	87.2	90.8
	R LASSO	11.0	3.3	0.030	0.127	0.130	86.2	91.0
	C LASSO	12.3	24.7	0.030	0.127	0.130	87.8	91.8
	TW LASSO	9.3	3.1	0.023	0.127	0.129	87.8	93.6
Yes	H LASSO	9.0	2.6	0.015	0.156	0.157	95.6	98.8
	R LASSO	6.9	2.0	0.010	0.157	0.158	95.8	98.8
	C LASSO	9.1	3.1	0.003	0.153	0.153	96.6	99.0
	TW LASSO	6.8	1.2	0.020	0.151	0.152	97.2	98.8

Back

Two-way cluster dependence

- **Assumption AHK** Random elements $W_{it} = (Y_{it}, X_{it}, U_{it})$ are generated by the underlying process:

$$W_{it} = \mu + h(\alpha_i, \gamma_t, \varepsilon_{it}), \quad \forall i \geq 1, t \geq 1,$$

where $\mu = E[W_{it}]$; h is unknown; vector components $(\alpha_i)_{i\geq 1}$, $(\gamma_t)_{t\geq 1}$, and $(\varepsilon_{it})_{i\geq 1, t\geq 1}$ are mutually independent; α_i is i.i.d across i, ε_{it} is i.i.d across i and t, and γ_t is strictly stationary.
  - Common in cluster-robust inference literature.
- **Assumption AR** (*beta*-mixing of $\{\gamma_t\}_{t>1}$)
  - A generalization of Aldous-Huber-Kallenberg (AHK) representation (Chiang et al., 2024, REStat).

Assumption

For some $s > 1$ *and* $\delta > 0$,

1. $E[X_{it}' U_{it}] = 0$, $E[\|X_{it}\|^{8(s+\delta)}] < \infty$, $E[\|U_{it}\|^{8(s+\delta)}] < \infty$.
2. *Either* $\Lambda_a \Lambda_a' > 0$ *or* $\Lambda_g \Lambda_g' > 0$, *and* $N/T \to c$ *as* $(N, T) \to \infty$ *for some constant* c.

Back

High Dimensionality from Flexible Modeling

- Suppose X is $k \times 1$. Let L^{τ} be $\tau-$th order polynomial transformation and let r denote the approximation error.
- Then, the high dimensionality is realized as follows:

model	sparse approx.	dim. of unknown param.
$Y = f(X) + U$	no approx.	∞,
$Y = L^{\tau}(X)\beta + r + U$	$\tau = 2$	$k^2/2 + 3k/2$
$Y = L^{\tau}(X)\beta + r + U$	$\tau = 3$	$k^3/6 + k^2 + 11k/6$

Back

Absolute Regularity

Let $\|\nu\|_{TV}$ denote the total variation norm of a signed measure ν on a measurable space (S, Σ) where Σ is a σ-algebra on S:

$$\|\nu\|_{TV} = \sup_{A \in \Sigma} \nu(A) - \nu(A^c)$$

Define the dependence coefficient of X and Y as:

$$\beta(X, Y) = \frac{1}{2}\|P_{X,Y} - P_X \times P_Y\|_{TV}$$

Assumption (Absolute Regularity of $\{\gamma_t\}_{t \geq 1}$)

The sequence $\{\gamma_t\}_{t \geq 1}$ is beta-mixing at a geometric rate:

$$\beta_\gamma(q) = \sup_{s \leq T} \beta\left(\{\gamma_t\}_{t \leq s}, \{\gamma_t\}_{t \geq s+q}\right) \leq c_\kappa exp(-\kappa q), \forall q \in \mathbb{Z}^+,$$

for some constants $\kappa > 0$ and $c_\kappa \geq 0$.

Back

Assumption (Approximate Sparse Model)

The unknown function f can be well-approximated by a dictionary of transformations $f_{it} = F(X_{it})$ where f_{it} is a $p \times 1$ vector and F is a measurable map, such that

$$f(X_{it}) = f_{it}\zeta_0 + r_{it}$$

where the coefficients ζ_0 and the approximation error r_{it} satisfy

$$\|\zeta_0\|_0 \leq s = o(N \wedge T), \; \|r_{it}\|_{2,NT} \equiv R = O_P\left(\sqrt{\frac{s}{N \wedge T}}\right).$$

Back

My Construction of Weights

- I consider the following choice of penalty level λ and penalty weights ω: for each $j = 1, ..., p$,

$$\lambda = C_\lambda \frac{NT}{(N \wedge T)^{1/2}} \Phi^{-1}\left(1 - \frac{\gamma}{2p}\right),$$

$$\omega_j = \max\{\omega_{a,j}, \omega_{e,j}\} + \max\{\omega_{g,j}, \omega_{e,j}\} - \min\{\omega_{a,j}, \omega_{g,j}, \omega_{e,j}\},$$

$$\omega_{a,j} = \tfrac{N \wedge V}{N^2} \textstyle\sum_{i=1}^{N} a_{i,j}^2, \quad \omega_{g,j} = \tfrac{N \wedge V}{T^2} \textstyle\sum_{b=1}^{B} \left(\sum_{t \in H_b} g_{t,j}\right)^2,$$

$$\omega_{e,j} = \tfrac{N \wedge V}{NT} \textstyle\sum_{i=1}^{N} \left(\sum_{t=1}^{T} e_{it,j}\right)^2.$$

- Extra Tuning Parameters: C_λ, γ, B.
- Feasible weights: $\hat{a}_{i,j} = \frac{1}{T}\sum_{t=1}^{T} f_{it,j}\widehat{V}_{it}$, $\hat{g}_{t,j} = \frac{1}{N}\sum_{i=1}^{N} f_{it,j}\widehat{V}_{it}$, and $\hat{e}_{it,j} = f_{it,j}\widehat{V}_{it} - \hat{a}_{i,j} - \hat{g}_{t,j} + E_{NT}[f_{it,j}\widehat{V}_{it}]$.

Back

Tuning Parameters

- Tuning parameters for λ: $C_\lambda = O(1)$ and $\gamma = o(1)$. In practice, $C_\lambda = 2$ and $\gamma = \log(p \vee N \vee T)$.
- Tuning parameters for ω: $B = \text{round}(T/h)$, $h = \text{round}(T^{1/5}) + 1$, and $H_b = \{t : h(b-1) + 1 \leq t \leq hb\}$

Back

Valid feasible weights: There exist l, u such that $l\omega_j^{1/2} \leq \widehat{\omega}_j^{1/2} \leq u\omega_j^{1/2}$, uniformly over $j = 1, ..., p$ where $0 < l \leq 1$ and $1 \leq u < \infty$ such that $l \to 1$. Back

- As we allow the dimension of f_{it} to be larger than the sample size, the empirical Gram matrix $M_f = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} f_{it}f_{it}'$ is singular.
- However, it turns out we only need its certain sub-matrices to be non-singular.

Assumption (Sparse Eigenvalues)

For any $C > 0$, there exists constants $0 < \kappa_1 < \kappa_2 < \infty$ such that with probability approaching one as $(N, T) \to \infty$ jointly,

$$\kappa_1 \leq \min_{\delta \in \Delta(m)} \delta' M_f \delta < \max_{\delta \in \Delta(m)} \delta' M_f \delta \leq \kappa_2,$$

where $\Delta(m) = \{\delta : \|\delta\|_0 = m, \|\delta\|_2 = 1\}$.

Back

Assumption (Regularity Conditions)

(i) $\log(p/\gamma) = o\left(T^{1/6}/(\log T)^2\right)$. *(ii) For some* $\mu > 1, \delta > 0$, $\max_{j\leq p} E[|f_{it,j}|^{8(\mu+\delta)}] < \infty$, $E[|V_{it}|^{8\mu+\delta)}] < \infty$. *(iii)* $\min_{j\leq p} E(a_{i,j}^2) > 0$, $\min_{j\leq p} E(g_{t,j}^2) > 0$, *and* $\min_{j\leq p} E\left[\left(\sum_{t=1}^{T} e_{it,j}\right)^2 |\{\gamma_t\}_{t=1}^{T}\right] > 0$ *almost surely.*

Back

Rate of Convergence in the Oracle Case

- Consider the sample mean estimator $\widehat{\theta} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} Y_{it}$, which can be decomposed as follows:

$$\widehat{\theta} - \theta_0 = \frac{1}{N}\sum_{i=1}^{N} a_i + \frac{1}{T}\sum_{t=1}^{T} g_t + \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} e_{it},$$

  where $a_i := \mathrm{E}[Y_{it} - \theta_0|\alpha_i], g_t := \mathrm{E}[Y_{it} - \theta_0|\gamma_t]$, and $e_{it} := Y_{it} - \theta_0 - a_i - g_t$.
- Under some regularity conditions, for each j, $\widehat{\theta}_j = O_P\left(\frac{1}{\sqrt{N \wedge T}}\right)$ and $\|\widehat{\theta} - \theta_0\|_2 = O_P\left(\sqrt{\frac{s}{N \wedge T}}\right)$.

Back

Panel-DML: Orthogonalized Moment Condition

- Let $\varphi(W_{it}; \theta, \eta)$ be an identifying moment condition:

$$E[\varphi(W_{it}; \theta_0, \eta_0)] = 0$$

  where W_{it} are random elements; θ are the low-dim. parameters of interest and η are nuisance parameters.

- Let $\psi(W_{it}; \theta, \eta)$ be a corresponding orthogonal moment condition such that

$$\begin{aligned} E[\psi(W; \theta_0, \eta_0)] &= 0, \\ \partial_\eta E[\psi(W; \theta_0, \eta_0)][\eta - \eta_0] &= 0. \end{aligned}$$

Back

Cross Fitting Validity

Lemma (Independent Coupling)

Consider the main sample $W(k,l)$ and auxiliary sample $W(-k,-l)$ for $k = 1, ..., K$ and $l = 1, ..., L$. Suppose Assumptions 1-2 hold for $\{W_{it}\}$. Then, if $\log N/T \to 0$ as $N, T \to \infty$, we can construct $\tilde{W}(k,l)$ and $\tilde{W}(-k,-l)$ such that:

- *They are independent of each other;*
- *They have the same marginal distribution as $W(k,l)$ and $W(-k,-l)$, respectively;*

and

$$Pr\left\{(W(k,l), W(-k,-l)) \neq \left(\tilde{W}(k,l), \tilde{W}(-k,-l)\right), \text{ for some } (k,l)\right\} = o(1)$$

Back

Assumption (Statistical Rates and Score Regularity)

For some positive sequence (Δ_{NT}) *that* $\Delta_{NT} \to 0$, *we have*

(i) *For each* (k, l), *the nuisance estimator* $\widehat{\eta}_{k,l}$ *belongs to the realization set* $\mathcal{T}_{NT}$ *with probability* $1 - \Delta_{NT}$, *where* $\mathcal{T}_{NT}$ *contains* η_0.

(ii) *For all* $i \geq 1$, $t \geq 1$, *and some* $q > 2$, *the following moment conditions hold:*

$$m_{NT} := \sup_{\eta \in \mathcal{T}_{NT}} \left(E_P \left\| \psi(W_{it}; \theta_0, \eta) \right\|^q \right)^{1/q} < \infty, \tag{3}$$

$$m'_{NT} := \sup_{\eta \in \mathcal{T}_{NT}} \left(E_P \left\| \psi^a(W_{it}; \eta) \right\|^q \right)^{1/q} < \infty. \tag{4}$$

Assumption (Statistical Rates and Score Regularity)

(iii) *The following conditions on the statistical rates r_{NT}, r'_{NT}, λ'_{NT} hold for all $i \geq 1$, $t \geq 1$:*

$$r_{NT} := \sup_{\eta \in \mathcal{T}_{NT}} \| E_P[\psi^a(W_{it}; \eta) - \psi^a(W_{it}; \eta_0)] \| \leq \delta_{NT},$$

$$r'_{NT} := \sup_{\eta \in \mathcal{T}_{NT}} \left(E_P \| \psi(W_{it}; \theta_0, \eta) - \psi(W_{it}; \theta_0, \eta_0) \|^2 \right)^{1/2} \leq \delta_{NT},$$

$$\lambda'_{NT} := \sup_{r \in (0,1), \eta \in \mathcal{T}_{NT}} \left\| \partial_r^2 E_P[\psi(W_{it}; \theta_0, \eta_0 + r(\eta - \eta_0))] \right\| \leq \delta_{NT}/\sqrt{N}.$$

Back

Assumption (Linear Orthogonal Scores)

For any $P \in \mathcal{P}_{NT}$, the following conditions hold:

(i) $\psi(W;\theta,\eta) = \psi^a(W,\eta)\theta + \psi^b(W,\eta),\ \forall\ W \in \mathcal{W}, \theta \in \Theta, \eta \in \mathcal{T}$.

(ii) $\psi(W;\theta,\eta)$ *satisfy the Neyman orthogonality conditions, or more generally, by a λ_{NT} near-orthogonality condition:* $\lambda_{NT} := \sup_{\eta \in \mathcal{T}_{NT}} \|\partial_r E[\psi(W;\theta_0,\eta_0 + r(\eta - \eta_0))]|_{r=0}\| \leq \delta_{NT}/\sqrt{N}$, *where $\mathcal{T}_{NT} \in \mathcal{T}$ is a nuisance realization set.*

(iii) *The map $\eta \to E_P[\psi(W_{it};\theta,\eta)]$ is twice continuously Gateaux-differentiable on $\mathcal{T}$.*

(iv) *The singular values of the matrix $A_0 := E_P[\psi^a(W_{it};\eta_0)]$ are bounded between a_0 and a_1.*

(v) *Either $\lambda_{min}[\Lambda_a\Lambda_a'] > 0$ or $\lambda_{min}[\Lambda_g\Lambda_g'] > 0$.*

Back

Variance Estimators

$$\widehat{V}_{CHS} = \widehat{A}^{-1}\widehat{\Omega}_{CHS}\widehat{A}^{-1\prime}, \qquad \widehat{V}_{DKA} = \widehat{A}^{-1}\widehat{\Omega}_{DKA}\widehat{A}^{-1\prime}$$

$$\widehat{\Omega}_{CHS} = \widehat{\Omega}_A + \widehat{\Omega}_{DK} - \widehat{\Omega}_{NW}, \quad \widehat{\Omega}_{DKA} = \widehat{\Omega}_A + \widehat{\Omega}_{DK}.$$

where $\widehat{A} = \frac{1}{KL}\sum_{k=1}^{K}\sum_{l=1}^{L}\frac{1}{N_k T_l}\sum_{i\in I_k, s\in S_l}\psi^a(W_{it};\widehat{\eta}_{kl})$, and

$$\widehat{\Omega}_A := \frac{1}{KL}\sum_{k=1}^{K}\sum_{l=1}^{L}\frac{1}{N_k T_l^2}\sum_{i\in I_k, t\in S_l, r\in S_l}\psi(W_{it};\widehat{\theta},\widehat{\eta}_{kl})\psi(W_{ir};\widehat{\theta},\widehat{\eta}_{kl})',$$

$$\widehat{\Omega}_{DK} := \frac{1}{KL}\sum_{k=1}^{K}\sum_{l=1}^{L}\frac{K/L}{N_k T_l^2}\sum_{t\in S_l, r\in S_l} k\left(\frac{|t-r|}{M}\right)\sum_{i\in I_k, j\in I_k}\psi(W_{it};\widehat{\theta},\widehat{\eta}_{kl})\psi(W_{jr};\widehat{\theta},\widehat{\eta}_{kl})',$$

$$\widehat{\Omega}_{NW} := \frac{1}{KL}\sum_{k=1}^{K}\sum_{l=1}^{L}\frac{K/L}{N_k T_l^2}\sum_{i\in I_k, t\in S_l, r\in S_l} k\left(\frac{|t-r|}{M}\right)\psi(W_{it};\widehat{\theta},\widehat{\eta}_{kl})\psi(W_{ir};\widehat{\theta},\widehat{\eta}_{kl})'.$$

where $k\left(\frac{m}{M}\right) = 1 - \frac{m}{M}$ is the Bartlett kernel and M is the bandwidth parameter.

Back

Asymptotic Normality without Cross-Fitting

- Under sparse approximation and Mundlak device, the (near) Neyman-orthogonal moment function is given by

$$\psi(W_{it};\theta,\eta) := (Z_{it} - f_{it}\zeta_0)\left(Y_{it} - f_{it}\beta_0 - \theta_0\left(D_{it} - f_{it}\pi_0\right)\right).$$

where f_{it} includes a constant and the polynomial transformation of $(X_{it}, \bar{X}_i, \bar{X}_t, \bar{D}_i, \bar{D}_t)$.

- **Theorem**: Under Assumptions AHK, generalized Mundlak device, regularity conditions and sparse approximation with $s = o\left(\frac{\sqrt{N \wedge T}}{\log(p/\gamma)}\right)$, $\|r_{it}^{\iota}\|_{NT,2} = o_P\left(\sqrt{\frac{1}{N \wedge T}}\right)$ for $l = Y, D$, as $N, T \to \infty$ and $N/T \to c$ where $0 < c < \infty$, the full-sample two-step estimator is asymptotically normal.

- consistent variance estimators using full sample Back

Assumption (Regularity Conditions for the Partial Linear Model)

(i) *A_0 is non-singular.*

(ii) *For any ϵ, $h_c(F, \epsilon)$ and $h_d(F, \epsilon)$ are invertible in F.*

(iii) *For some $\mu > 1, \delta > 0$, $\max_{j \leq p} \mathrm{E}[|f_{it,j}|^{8(\mu+\delta)}] < \infty$ and $\mathrm{E}[|V_{it}^l|^{8(\mu+\delta)}] < \infty$ for $l = g, D, Y, Z$.*

(iv) *Either $\lambda_{min}[\Sigma_a] > 0$ or $\lambda_{min}[\Sigma_g] > 0$; $\min_{j \leq p} E[a_{i,j}^l]^2 > 0$, $\min_{j \leq p} E[g_{t,j}^l]^2 > 0$, $\min_{j \leq p} E\left[\left(\sum_{t=1}^T e_{it,j}^l\right)^2 |\{\gamma_t\}_{t=1}^T\right] > 0$ almost surely, for $l = D, Y, Z$.*

(v) $\log(p/\gamma) = o\left(T^{1/6}/(\log T)^2\right)$.

(vi) *The feasible penalty weights $\widehat{\omega}_l$ satisfy the* condition *for $l = D, Y, Z$.*

(viii) *sparse eigenvalues* condition.

Back

Variance estimators using full sample

Theorem

Suppose assumptions for Theorem holds for $P = P_{NT}$ *for each* (N, T) *with* $r_{it}^{D} = r_{it}^{Y} = 0$ *a.s., and* $M/T^{1/2} = o(1)$. *Then,* $(N, T) \to \infty$ *and* $N/T \to c$ *where* $0 < c < \infty$,

$$\widehat{V}_{\mathrm{CHS}} = V + o_P(1),$$

$$\widehat{V}_{\mathrm{DKA}} = \widehat{V}_{\mathrm{CHS}} + o_P(1).$$

Back

Government Spending Multiplier: Cross-Fitting Method

Table 3: Estimates of the open economy relative multiplier from the extended model.

(1) Cross-Fitting	(2) Poly. Trans.	(3) Param. Gen.	(4) First Stage	(5) Z: Param. Ave. Sel.	(6) $\widehat{\theta}$	(7) CHS s.e.	(8) DKA s.e.
Yes	None	7	H LASSO	2.0	1.28	1.73	2.00
			C LASSO	2.0	1.32	1.75	2.03
			TW LASSO	2.6	1.18	1.77	2.05
Yes	2nd	35	H LASSO	5.2	1.12	2.18	2.52
			C LASSO	5.8	1.46	1.95	2.24
			TW LASSO	4.1	1.20	1.42	1.70
Yes	3rd	119	H LASSO	8.3	1.81	3.17	3.47
			C LASSO	6.5	1.25	1.59	1.91
			TW LASSO	5.3	1.50	1.18	1.44

Back

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